Corruption and production: a policy analysis.

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Abstract
This paper analyzes the relation existing between corruption, monitoring and output in an economy. By solving a dynamic game we prove that equilibrium output is a non-linear upper-hemicontinuous function (MP function) of the monitoring level implemented by the State on corruption, presenting 3 different equilibrium scenarios. According to our model, we analyze the optimal strategy depending on the policy objective of the State and we prove that if the State is budget constrained the optimal policy can lead the economy to an equilibrium with widespread corruption and maximum production.

JEL codes: C65, C73, D73, E23
Keywords: Corruption, Dynamic game, Equilibrium production, Policy analysis

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1 Introduction.

In this work we construct a theoretical model explaining the relation existing between corruption, monitoring and production in an economy. By solving a 3–period dynamic game within the principal–agent scheme, we derive a function that associates to each monitoring level on the corruption implemented by the State a unique total equilibrium output produced in the economy under the hypothesis placed.

Several authors, like Barreto (2000), Del Monte and Papagni (2001) and Acemoglu and Verdier (1998,2000), studied models where corruption emerges from a market failure and found–out that the relation between corruption and economic growth is non–linear.

Precisely, while Acemoglu and Verdier (1998,2000) determine the optimal corruption level (due to the presence of a trade–off between a public failure and a market one) in a wage efficiency model,\(^1\) Barreto (2000) showed that corruption can be “efficiency enhancing” since it reduces bureaucracy inefficiencies. Finally Del Monte and Papagni (2001) construct a model where corruption reduces public infrastructure quality that are necessary for production and hence it has a negative indirect impact on economic growth.

We reach similar conclusions since we derive a non–linear relation between corruption and output and we prove that greater corruption can be associated to greater production. Differently from the previous works, in our model corruption is both the consequence of the low monitoring level implemented by the State, and the existence of an entrepreneur return eventually used to pay a bribe and, finally, the monopolistic and discrestional bureaucrat power.

Differently form Barreto (2000), most economic researchers attach great importance to corruption because of its significant negative impact on investment. For instance Mauro (1995,1998) showed that a country that improves its standing on corruption will experience an increase in its investment and GDP growth rate. Furthermore Campos et al. (1999) pointed out on the predictability of corruption by showing that the negative impact of the corruption on investment is lesser for those corruption regimes that are predictable.

Our conclusions are partially different from the previous ones since we prove that with a low monitoring level, high corruption can lead to great output.

The modern analysis of corruption (starting from the works by Rose–Ackerman, 1975, 1978) places such phenomenon within a principal–agent

\(^1\)See also Besley and McLaren (1993) and Van Rijckeghem and Weber (1997) who proposed alternative bureaucrat payment schemes in the presence of corruption.
scheme. In fact the existence of corruption is subject to an agency relationship between an individual in charge of making decisions (the agent) and the owner of the interests (principal) he represents. A third party (in this model the entrepreneur) should then be involved to influence the agent’s discretionary decisions to his own benefit, upon the payment of a bribe. Following the assumption of most recent works (e.g. Shleifer and Vishny, 1993) the agency relationship between the bureaucrat (agent) and the Government (principal) will not be analyzed, but focus will only be made on the possible relation between the bureaucrat and the entrepreneur (third party) in order to better highlight the mechanisms through which such relations develop as well as the effects of such behaviors on the economy. In our model the State is an external agent who controls the bureaucrats’ and entrepreneurs’ behavior by fixing the monitoring level.

Here we consider the model studied in Coppier and Piga (2004) while assuming the more realistic hypothesis that entrepreneur can diversify production both investing in the traditional as well as in the modern sector of the economy. Furthermore, here we are replacing the hypothesis of constant marginal returns to capital with the one of decreasing marginal returns to capital in order to compare the achieved results. The approach we used is mainly theoretical: we first formalize and solve a 3-period dynamic game describing the model, second we find out the equilibrium output at each scenario and finally we prove the properties of the monitoring-production (MP) function and we discuss the optimal policy conclusions. The MP function is non-linear and upper hemicontinuous furthermore it is decreasing for low values of the monitoring level, it means that, differently from Mauro (1998), more corruption could be associated with more production. In fact at low monitoring levels the economy experiences widespread corruption associated with greater output than the one in the case of intermediate monitoring level. Finally only sufficiently high monitoring level guarantees maximum production without corruption. Consequently, since monitoring is costly, if the State is budget constrained it may find it convenient to accept corruption implementing a low monitoring level and obtaining an intermediate output level.

The paper is organized as follows. In Section 2 we first present the model and then we formalize and solve the dynamic game describing the model. In Section 3 we demonstrate the non-linear relation between the monitoring

\(^2\)While Coppier and Piga (2004) derived a discontinuous piecewise-constant function since they assume a linear technology.

\(^3\)The implication resulting from our model is also confirmed by data for the Italian case in the 1971-1996, see Coppier and Piga, 2004.
level of bureaucrats by the State, corruption and equilibrium output and we describe its properties. In Section 4 we discuss policy considerations and we compare different optimal policies related to the existence of a budget constraint faced by the State. We conclude in Section 5.

2 The model.

Let us consider an economy producing a single homogeneous good. There are two distinct categories of individuals in such an economy. First the bureaucrats, who cannot invest in the production activity, and second the entrepreneurs, who may invest their total capital available both in the modern sector and in the traditional one. There is a continuum of bureaucrats and entrepreneurs and their number is normalized to one for both categories. A third agent is the State who controls entrepreneurs' and bureaucrats' behavior in order to weed out or reduce corruption.

The production function of the tradeable good only depends on the capital \( k > 0 \) available to the entrepreneur which can be invested in the modern sector or in the traditional one. Let \( k_M \) and \( k_T \) be the capital amounts respectively invested in the modern or in the traditional sector \((k_M, k_T \geq 0)\). Output is obtained through technologies with decreasing marginal returns to capital, that is

\[
y = k_M^\alpha + k_T^\beta
\]

where \( k_M + k_T = k \) (all the capital amount is invested) and \( 0 < \beta < \alpha < 1 \). We assume both that \( \alpha, \beta \in (0,1) \) because of the decreasing marginal returns to capital hypothesis, and also that \( \beta < \alpha \) since the returns in the traditional sector are less than the ones in the modern sector.\(^4\)

The entrepreneur who wants to invest a positive capital amount \( k_M \) in the modern sector needs to obtain a licence from the bureaucrat to access the technology. In order to obtain such a licence he has to submit a project to the Public Administration whose cost is a proportion \( s > 0 \) of the capital \( k_M \) the entrepreneur wants to invest in the modern sector, then the submission cost is \( sk_M \). We assume \( k_M^\alpha + k_T^\beta - (k_T + k_M)^\beta > sk_M \) for investment be convenient in the modern sector.\(^5\)

\(^4\)In our model corruption transactions are due to the existence of a non–competitive sector (the modern one) that generates incomes for entrepreneurs that can be used to pay a bribe to the bureaucrat.

\(^5\)If \( k_M^\alpha + k_T^\beta - (k_T + k_M)^\beta \leq sk_M \) then the submission cost exceeds the return gains between the two sectors.
Once the entrepreneur presents the project to the bureaucrat, the latter may decide to be honest or corrupt by issuing the licence in exchange for a bribe.\(^6\) Let \(b^d \geq 0\) be the bribe asked to issue the licence.\(^7\) The entrepreneur could refuse the payment of the bribe or accept the bribe asked or open a negotiation on the bribe with the bureaucrat. The bureaucrat is assumed to have monopolistic and discretion power, that is, he is the only one who may issue the licence or refuse it without any explanation.

The State has a control role on the behavior of entrepreneurs and bureaucrats. Let \(q \in [0, 1]\) be the monitoring level implemented by the State such that there is a probability \(q\) of being detected in an extortion. In this paper we consider \(q\) as exogenous.

The punishment for an act of corruption is a proportional cost (monetary, moral or criminal) of the submitted project capital amount \(k_M\) both for entrepreneur and bureaucrat as in Rose–Ackerman (1999). Let \(mk_M\) be the punishment for the detected bureaucrat and \(ck_M\) the one for the detected entrepreneur, where \(m, c > 0\). We also assume \(m \geq c\) since the bureaucrat has the discretion power to demand the bribe and the monopolistic power to issue the licence.\(^8\)

The aim of this work is to determine the optimal level of invested capital in each productive sector and the output level of the economy under corruption. In order to find out the relation between corruption, monitoring and output, we proceed with the formalization and solution of the dynamic game describing the model.

\section*{2.1 Dynamic game. Description and solution.}

Given the model just described, the economic problem can be formalized by the following three-period dynamic game.

(1) At stage one of the game the entrepreneur decides the capital amount to be invested in the modern sector \(k_M\) and consequently \(k_T = k - k_M\).

(1.1) If \(k_M = 0\), all the capital is invested in the traditional sector.

Let \(W > 0\) be the wage for the bureaucrat, then the payoff vector for

\(^6\)Differently from Shleifer and Vishny (1993) in our model we assume that corruption is without theft.

\(^7\)We are assuming the bribe asked is constant and does not depend on \(k_M\). However, it can be shown that our results would not change by placing \(b^d = bk_M\), being \(b \geq 0\).

\(^8\)As underlined in Rose–Ackerman (1999) in several countries bribe payers are treated more leniently than recipients.
bureaucrat and entrepreneur is

$$\pi_1 = (W, (k_T + k_M)\beta) \quad (2)$$

being $k_T = k$. The game ends.

(1.2) Otherwise, let $k_M \neq 0$. Then the entrepreneur submits the project and pays the cost $sk_M$. The game continues to stage two.

(2) At stage two the bureaucrat decides the amount of the asked bribe $b^d$ to issue the licence.

(2.1) If $b^d = 0$ no bribe is asked, then the payoff vector for bureaucrat and entrepreneur is

$$\pi_2 = (W, k_M^\alpha + k_T^\beta - sk) \quad (3)$$

being $k_M + k_T = k$, $k_M \neq 0$. The game ends.

(2.2) Otherwise, let $b^d > 0$ be the positive bribe asked by the bureaucrat. The game continues to stage three.

(3) At stage three the entrepreneur decides whether to negotiate the bribe or refuse it.

(3.1) If the entrepreneur refuses the bribe, then the payoff vector is given by

$$\pi_3 = (W, (k_T + k_M)^\beta - sk_M) \quad (4)$$

being $k_T + k_M = k$. Then the game ends.

(3.2) Otherwise the negotiation starts. Let $b^{NB}$ be the final equilibrium bribe associated to the Nash solution to a bargaining game that is the result of the negotiation. Then, given the monitoring level on the corruption pursued by the State $q$, the payoff vector is

$$\pi_4 = \left(W - qmk_M + (1 - q)b^{NB}, k_M^\alpha + k_T^\beta - (s + qc)k_M - (1 - q)b^{NB}\right) \quad (5)$$

being $k_T + k_M = k$, $k_M \neq 0$. The game ends.

We first determine the equilibrium bribe $b^{NB}$ (see Appendix A for the proof).

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\(^9\text{See Binmore et al. (1986).}\)
Proposition 2.1. Let \( q \neq 1 \). Then there exists a unique positive equilibrium bribe \( b^{NB} \), the Nash solution to a bargaining game in the last subgame, given by

\[
b^{NB} = \frac{k_M^\alpha + k_T^\beta - (k_T + k_M)^\beta + q(m - c)k_M}{2(1 - q)}.
\]  

(6)

Notice that the entrepreneur pays half of the expected surplus\(^{11}\) to the bureaucrat as a bribe.

Having determined the equilibrium bribe \( b^{NB} \) in (6), the payoff vector in (5) can be written in the following way

\[
\pi_4 = \left( W + \frac{k_M^\alpha + k_T^\beta - (k_T + k_M)^\beta - q(m + c)k_M}{2}, \frac{k_M^\alpha + k_T^\beta + (k_T + k_M)^\beta - q(m + c)k_M - 2sk_M}{2} \right).
\]  

(7)

In Appendix B we solve the dynamic game by using the backward induction method, starting from the last stage of the game. We proved the following proposition.

Proposition 2.2. Let \( \frac{k_M^\alpha + k_T^\beta - (k_T + k_M)^\beta}{(m + c)k_M} = q_1 < 1 \) and \( \frac{k_M^\alpha + k_T^\beta - (k_T + k_M)^\beta - 2sk_M}{(m + c)k_M} = q_2 > 0 \).

(a) If \( q \in [0, q_2] \) then the payoff vector is

\[
\pi_4 = \left( W + \frac{k_M^\alpha + k_T^\beta - (k_T + k_M)^\beta - q(m + c)k_M}{2}, \frac{k_M^\alpha + k_T^\beta + (k_T + k_M)^\beta - q(m + c)k_M - 2sk_M}{2} \right);
\]

(b) if \( q \in (q_2, q_1) \) then the payoff vector is

\[
\pi_1 = (W, (k_T + k_M)^\beta);
\]

(c) if \( q \in [q_1, 1] \) then the payoff vector is

\[
\pi_2 = (W, k_M^\alpha + k_T^\beta - sk_M);
\]

\(^{10}\)If \( q = 1 \) there is no incentive for the bureaucrat to demand the bribe.

\(^{11}\)The expected surplus is the difference between the return if investing in both sectors net of the entrepreneur’s expected cost and plus the bureaucrat’s expected cost, if they are both detected in an act of corruption.
The previous proposition 2.2 shows that by the backward solution of the dynamic game we obtain three perfect Nash equilibria in the sub-games, depending on the parameter values, that could be summarized as follows:

- if \( q \in [0,q_2] \), the entrepreneur submits the project at stage one, the bureaucrat asks for a positive bribe at stage two, and finally, at stage three, the entrepreneur starts a negotiation that ends at the equilibrium bribe \( b^{NB} \) given by (6). The game ends and the payoff vector is given by (7). This equilibrium is with corruption. In such a case the difference in profits between the modern sector and what is obtained from the traditional sector is such as to make up for the expected cost of one’s corruption, of the bureaucrat’s corruption and for the project submission cost. Thus the surplus to be shared between the entrepreneur and the bureaucrat allows for a negotiation. The outcome is the bribe corresponding to the Nash solution to a bargaining game;

- if \( q \in (q_2,q_1) \), at stage one the entrepreneur does not submit the project. The game ends and the payoff vector is given by (2). In such a case the entrepreneur knows that what he would obtain from submitting the project is less than what he would obtain by investing in the traditional sector, therefore he will not submit the project. In such a case there is no corruption but all the entrepreneurs invest in the traditional sector;

- if \( q \in [q_1,1] \), the entrepreneur submits the project at stage one and the bureaucrat does not ask any positive bribe. The game ends and the payoff vector is given by (3).

### 3 Equilibrium output under corruption.

According to the solution of our dynamic game, we now consider the maximization problem faced by the entrepreneur in order to determine the optimal capital level to be invested in the modern sector and in the traditional one.

To simplify the analysis, in what follows we place \( k = 1 \) so that what we must determine are the capital shares invested in both sectors. The following proposition states the optimal capital amount invested in the modern sector (thus, also the amount invested in the traditional sector) corresponding to the three regimes previously determined (see appendix C for the proof).

**Proposition 3.1.** Let \( k = 1 \) and assume \( 0 < q_2 \) and \( q_1 < 1 \).

(a) If \( q \in [0,q_2] \) a unique equilibrium amount of capital invested in the modern sector \( k^*_M = \phi(q) \) exists such that \( \phi(q) \) is continuous and \( \phi'(q) < 0 \).
(b) If \( q \in (q_2, q_1) \) the amount of capital invested in the modern sector is \( k_M = 0 \), so \( k_T = 1 \).

(c) If \( q \in [q_1, 1] \) the amount of capital invested in the modern sector is \( k_M^* \in (0, 1) \), thus also \( k_T^* \in (0, 1) \).

Notice that with widespread corruption (that is \( q \leq q_2 \)) the amount of capital invested in the modern sector is a decreasing function of the monitoring level while in the other cases it does not depend on the monitoring level.

Now we want to represent the qualitative graph of the equilibrium amount of capital invested by the entrepreneur in the modern sector, \( k_M^* \), with respect to the monitoring level implemented by the State on corruption, \( q \). Given proposition 3.1, we find the following function

\[
k_M^* = \omega(q) = \begin{cases} 
\phi(q) & q \in [0, q_2] \\
0 & q \in (q_2, q_1) \\
\kappa & q \in [q_1, 1]
\end{cases}
\]

(8)

\( \omega(q) \) associates a unique equilibrium \( k_M^* \in [0, 1] \) to each \( q \in [0, 1] \).

In order to investigate the geometric properties of \( \omega(q) \) note that such a function represents the solution of the first order condition to the maximization problem for each of the three situations depending on the values of \( q \) (see appendix C for the details).

More precisely, let \( g(k_M) = \alpha k_M^{\alpha-1} - \beta (1-k_M)^{\beta-1} \), \( \gamma_1 = q(m+c) + 2s > 0 \) and \( \gamma_2 = s > 0 \). Then the function \( k_M^* = \omega(q) \) is the solution for the following equations:

(i) \( g(k_M) = \gamma_1 \) if \( q \in [0, q_2] \),

(ii) \( k_M = 0 \) if \( q \in (q_2, q_1) \),

(iii) \( g(k_M) = \gamma_2 \) if \( q \in [q_1, 1] \).

As we have proven in appendix C, the solution of (i) is the continuous decreasing function \( k_M^* = \phi(q) \). Furthermore, if \( q = 0 \) we have to solve \( g(k_M) = 2\gamma_2 \); let \( k_M' \) be the solution to such an equation; if \( q = q_2 \) then \( k_M = \pi^1 \), so the solution of the maximization problem faced by entrepreneur is \( k_M = 0 \) as in (ii). Note that \( \omega(q) \) is continuous in \( q_2 \).

Furthermore \( \omega(q) \) is constant and equal to zero \( \forall q \in (q_2, q_1) \) as stated in (ii).
Finally if \( q \in [q_1, 1] \) the solution of the equation in \((iii)\) is a constant value of \( k_M \). We call such a value \( \kappa \); it is dependent on \( s \). This equilibrium value is positive, as we prove in appendix C and \( \kappa > k'_M \). It is straightforward to prove that function \( \omega(q) \) is upper–hemicontinuous. In figure 1 we present the graphic solution of the previous equations and the qualitative graph of \( k^*_M = \omega(q) \).

Now we may discuss the qualitative properties that the output equilibrium level has with respect to the State monitoring. In appendix D we prove the following proposition 3.2.

**Proposition 3.2.** Let \( k = 1 \) and assume \( 0 < q_2 \) and \( q_1 < 1 \).

(a) If \( q \in [0, q_2] \) then a unique equilibrium output level \( y^* = \psi(q) \) exists such that \( \psi(q) \geq 1 \) is continuous and \( \psi'(q) < 0 \);

(b) if \( q \in (q_2, q_1) \) then the equilibrium output level is \( y^* = 1 \);

(c) if \( q \in [q_1, 1] \) then the equilibrium output level is \( y^* = \varphi \) where \( \varphi > 1 \).

The function that associates the unique equilibrium output to each monitoring level on corruption implemented by the State is upper-hemicontinuous and is given by the following

\[
y^* = \Omega(q) = \begin{cases} 
\psi(q) < \varphi & q \in [0, q_2] \\
1 & q \in (q_2, q_1) \\
\varphi > 1 & q \in [q_1, 1]
\end{cases}
\]  

Figure 1: (a) Solution to equations \((i)\) and \((iii)\); (b) qualitative representation of \( k^*_M = \omega(q) \).
We call such a function MP (monitoring–production). The qualitative graph of the MP function is in figure 2.

Figure 2: The MP-function: equilibrium production level with respect to the monitoring of corruption.

4 Policy considerations.

Let us consider the MP function just derived. Then the State could choose to achieve one of the following (first) policy objectives: maximize output, weed out corruption or minimize monitoring costs. So we analyze the following three cases:

- if the State wants to maximize production without considering the monitoring costs and the corruption level, it has to set the monitoring level at \( q \geq q_1 \). Notice that \( \forall q \geq q_1 \) corruption is absent, hence, since monitoring is expensive, the optimal policy is to set the monitoring level at \( q = q_1 \) with consequent output level being \( y^* = \varphi > 1 \);

- if the State wants to weed–out corruption it has to set a monitoring level \( q \geq q_2 \) so, without corruption, only two different equilibrium outputs could be reached that are \( y^* = 1 \) and \( y^* = \varphi > 1 \). If the second policy objective is to maximize production then the State has to set \( q \geq q_1 \), and consequently \( q = q_1 \) since monitoring is expensive, so \( y^* = \varphi > 1 \). If its second policy objective is to minimize monitoring costs then the State will set \( q = q_2 \) and consequently \( y^* = 1 \);
• if the State wants to minimize monitoring costs it has to set the monitoring level \( q = 0 \) and the equilibrium output that will be reached is \( y^* = y(k'_M) \in (1, \varphi) \); such an objective is not compatible with corruption elimination.

The previous considerations are summarized in table 1.

<table>
<thead>
<tr>
<th>No budget constraint</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Objectives</td>
<td>2nd: max production</td>
<td>2nd: no corruption</td>
<td>2nd: min costs</td>
</tr>
<tr>
<td>1st: max production</td>
<td>( q = q_1 \Rightarrow y^* = \varphi &gt; 1 )</td>
<td>( q = q_1 \Rightarrow y^* = \varphi &gt; 1 )</td>
<td>( q = q_1 \Rightarrow y^* = \varphi &gt; 1 )</td>
</tr>
<tr>
<td>1st: no corruption</td>
<td>( q = q_1 \Rightarrow y^* = \varphi &gt; 1 )</td>
<td>( q \geq q_2 \Rightarrow y^* = {1, \varphi} )</td>
<td>( q = q_2 \Rightarrow y^* = 1 )</td>
</tr>
<tr>
<td>1st: min costs</td>
<td>( q = 0 \Rightarrow y^* \in (1, \varphi) )</td>
<td>( \vdots \vdots \vdots )</td>
<td>( q = 0 \Rightarrow y^* \in (1, \varphi) )</td>
</tr>
</tbody>
</table>

Table 1: First and second policy objective and the associated monitoring levels.

Notice that a unique optimal policy according to the MP function does exist, having the properties “maximum production– no corruption” at the same time, and that in order to reach such a point it needs public investment on the struggle against corruption such that \( q = q_1 \). Hence, by fixing a monitoring level \( q = q_1 \), the State can reach its first policy objective “maximize production” without corruption and at minimum cost.

On the contrary, if the first policy objective is “no corruption” then the State cannot reach the two second objectives “minimize costs” and “maximize production” at the same time.

Finally, the unique optimal policy in order to minimize monitoring costs is achieved by fixing \( q = 0 \); this point determines the unique viable production level with widespread corruption.

The equilibrium points just described are all viable, if and only if, the State disposes of sufficient resources such that the monitoring level \( q = q_1 \) can be achieved.

However monitoring is expensive. Consequently there could be different reasons why \( q = q_1 \) would not viable. For example the State faces a budget constraint or it prefers to spend its resources in other ways.

Let \( q^* \) be the maximum monitoring level the State can implement according to its budget constraint.

Then it is straightforward to see that if \( q^* \geq q_1 \), all the equilibria illustrated in table 1 are viable since the constraint is not binding.

On the contrary, if \( q_2 \leq q^* < q_1 \), we have the following scenario:
• in order to maximize production the monitoring level must be set at \( q = 0 \); at the same time costs are minimized but corruption cannot be avoided; the equilibrium output that will be reached is \( y^* = y(k'_M) \in (1, \varphi) \);

• in order to weed out corruption the monitoring level must be set at \( q_2 \leq q \leq q^* \) which implies \( y^* = 1 \); since production is constant through the interval \([q_2, q^*]\) and since monitoring is costly, the optimal policy is to fix \( q = q_2 \);

• in order to minimize monitoring costs the monitoring level must be set at \( q = 0 \) and the equilibrium production that will be reached is \( y^* = y(k'_M) \in (1, \varphi) \) with corruption.

So we can conclude that the State has only two different viable optimal policies as illustrated in table 2.

<table>
<thead>
<tr>
<th>Budget constraint s.t. ( q_2 \leq q^* &lt; q_1 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Objectives</td>
</tr>
<tr>
<td>1\textsuperscript{st}: max production</td>
</tr>
<tr>
<td>1\textsuperscript{st}: no corruption</td>
</tr>
<tr>
<td>1\textsuperscript{st}: min costs</td>
</tr>
</tbody>
</table>

Table 2: First and second policy objectives and the associated monitoring level with budget constraint.

In such a case the equilibrium “maximum production–no corruption” is not viable since the unique optimal policy is \( q = 0 \) corresponding to “maximum production–minimum cost” characterized by widespread corruption. Notice that this optimal policy is symmetric since, at the same time, it is also the solution of the policy objective “minimum cost–maximum production”. This argument shows that if the State is not interested in weeding out corruption then the optimal strategy is always \( q = 0 \). Otherwise, if the first policy objective is to avoid corruption, the optimal strategy is always \( q = q_2 \), which implies no investment in the modern sector with a consequent minimum output level.

Finally, if \( q^* < q_2 \) then the State cannot weed out corruption. Consequently, the optimal policy is to set \( q = 0 \) since it minimizes monitoring costs and maximizes production at the same time.

The first and second policy objectives chosen by the State depend on the weight it assigns to each of the three policy objectives we described, based
on its cost–benefit analysis. The model we construct proves that, if the State is budget constrained, the equilibrium with corruption could be desirable as the result of an optimal policy since it maximizes production and minimizes costs.\footnote{Similar conclusion in the case of asymmetry information are in Coppier and Piga (2004) where they considered the effect of a reform.}

The original result of the model we studied is that there could be cases where the State has convenience to accept corruption in order to maximize output.

## 5 Conclusions

In this paper we analyzed the relation existing between corruption, monitoring and production in an economy where entrepreneurs can invest capital both in the traditional as well as in the modern sector considering a non-linear technology with decreasing marginal returns.

We solved a 3–period dynamic game and we proved that equilibrium output is a non-linear upper-hemicontinuous function (MP function) of the monitoring level implemented by the State on the corruption presenting 3 different equilibrium scenarios. At low monitoring levels the economy experiences widespread corruption with intermediate decreasing output levels, at intermediate monitoring levels corruption is absent but low output level is achieved, finally at high monitoring levels no corruption occurs and high output level is recorded.

According to the function we derived, we discussed the optimal monitoring level implemented by the State in a normative perspective. By fixing the first and second policy objectives (maximizing production, minimizing costs and weeding out corruption, respectively), we distinguished between different optimal strategies depending on the existence of a budget constraint faced by the State. In particular we proved that if the State faces a budget constraint such that it cannot reach a sufficient monitoring level, then the optimal policy could be accept widespread corruption in order to maximize production at a minimum cost.

So in our paper we assessed that, contrary to previous works (see Mauro, 1995 and Shleifer and Vishny, 1993, among others), greater production could be associated to greater corruption so we proved that the equilibrium point with widespread corruption can be chosen as the result of an optimal policy implemented by the State.
A Appendix

Proof of proposition 2.1.

Proof. We refer to the bureaucrat payoff by a superscript $(1)$ and to the entrepreneur payoff by a superscript $(2)$: they represent respectively the first and second element of the payoff vector $\pi_i, i = 1, 2, 3, 4,$ placed in $(2), (3), (4), (5)$. Let $\pi_\Delta = \pi_4 - \pi_3 = (\pi_\Delta^{(1)}, \pi_\Delta^{(2)})$ be the vector of the differences in the payoffs between the case of agreement about the bribe (5) and that in the case of disagreement (4) for the bureaucrat and the entrepreneur. The bribe $b_{NB}^{NB}$ associated to the Nash solution to a bargaining game is the solution of the following maximum problem

$$\max_{b \in \mathbb{R}^+} (\pi_\Delta^{(1)} \cdot \pi_\Delta^{(2)})$$

in formula

$$\max_{b \in \mathbb{R}^+} (-qmk_M + (1 - q)b) \left( k_M^\alpha - qck_M - (1 - q)b + k_T^\beta - (k_T + k_M)^\beta \right)$$

that is the maximum of the product between the elements of $\pi_\Delta$. Since the objective function is concave with respect to $b$, a sufficient condition for $b$ being a maximum is the first order condition $\frac{\partial (\pi_\Delta^{(1)} \cdot \pi_\Delta^{(2)})}{\partial b} = 0 \Rightarrow$

$$\left( k_M^\alpha - qck_M - (1 - q)b + k_T^\beta - (k_T + k_M)^\beta \right) - (-qmk_M + (1 - q)b) = 0 \Rightarrow$$

$$k_M^\alpha + q(m - c)k_M - 2(1 - q)b + k_T^\beta - (k_T + k_M)^\beta = 0 \Rightarrow$$

$$b_{NB}^{NB} = \frac{k_M^\alpha + k_T^\beta - (k_T + k_M)^\beta + q(m - c)k_M}{2(1 - q)}$$

that is the unique equilibrium bribe in the last subgame, $\forall q \neq 1$.

Since $m - c \geq 0$, then $b_{NB}^{NB} > 0$.

\[\square\]

B Appendix

Proof of Proposition 2.2.

(3) At stage three the entrepreneur negotiates the bribe if and only if

\[
\frac{\pi_1^{(2)}}{\pi_3^{(2)}} > k_M^\alpha + k_T^\beta - s k_M - q c k_M - (1-q)b^{NB} > (k_T + k_M)^\beta - s k_M
\]

(see (5) and (4)) that is the entrepreneur’s payoff if he negotiates is greater then his payoff if he refuses. Since under perfect information hypothesis the entrepreneur knows the final equilibrium bribe \(b^{NB}\), given by (6), then we substitute this value in the previous inequality and, by simplification, we obtain

\[
k_M^\alpha + k_T^\beta - (k_T + k_M)^\beta - q (m + c) k_M > 0
\]

that is verified \(\forall q < \frac{k_M^\alpha + k_T^\beta - (k_T + k_M)^\beta}{(m + c) k_M} = q_1\).\(^{13}\)

Notice that in order to have an admissible probability set, \(q_1\) must be greater or equal to 0. This inequality is always verified since \(\forall k_M, k_T > 0\) we have \((k_T + k_M)^\beta \leq k_M^\beta + k_T^\beta < k_M^\alpha + k_T^\beta\) being \(\alpha > \beta\).

Furthermore, if \(q_1 < 1\) then a probability set having a positive measure such that the entrepreneur has no convenience to negotiate the bribe exists. Notice that

\[
q_1 < 1 \iff k_M^\alpha + k_T^\beta - (k_T + k_M)^\beta - (m + c) k_M < 0.
\]

Let \(q_1 < 1\) then if \(q < q_1\) the entrepreneur negotiates the bribe, otherwise if \(q \geq q_1\) he refuses the bribe.\(^{14}\)

(2) Going up the decision-making tree, at stage two the bureaucrat decides whether to ask a positive bribe.

- Let \(q \geq q_1\) then the bureaucrat knows that the entrepreneur will not accept any positive bribe so he will be honest and will pursue the licence without any bribe.

- Let \(q < q_1\) then the bureaucrat knows that if he asks a positive bribe then the entrepreneur will accept the negotiation and the final bribe will be \(b^{NB}\) given by (6). Then at stage two the bureaucrat asks a bribe if and only if

\[
\frac{\pi_1^{(2)}}{\pi_2^{(2)}} > W - q m k_M + (1-q)b^{NB} > W
\]

\(^{13}\)Notice that \(k_M\) must be different from 0. This condition is verified because we are studying stage three of the game so the entrepreneur has already submitted the project to invest a positive capital amount in the modern sector.

\(^{14}\)Otherwise if \(q_1 \geq 1\) then the entrepreneur always negotiates the bribe.
(see (5) and (3)) that is the bureaucrat’s payoff if asking a positive bribe is greater than his payoff if not asking the bribe. By substituting $b^{NB}$ given by (6) in the previous inequality and simplifying the previous inequality, we obtain

$$k^\alpha_M + k^\beta_T - (k_T + k_M)^\beta - q(m + c)k_M > 0$$

that holds $\forall q < q_1$. So we can conclude that if $q < q_1$ then the bureaucrat asks the bribe $b^{NB}$ that the entrepreneur accepts.

(1) At stage one the entrepreneur has to decide whether to present the project.

- Let $q \geq q_1$ then the entrepreneur knows that if he presents a project no bribe will be asked. So he will present the project if and only if

$$\pi^{(2)}_2 > \pi^{(2)}_1 \Rightarrow k^\alpha_M - sk_M + k^\beta_T > (k_T + k_M)^\beta.$$

(see (3) and (2)) that is if $k^\alpha_M + k^\beta_T - (k_T + k_M)^\beta > sk_M$. The previous inequality is always verified for hypothesis.

- Let $q < q_1$ then the entrepreneur knows that the bureaucrat will ask the bribe $b^{NB}$ that he will accept. So, at stage one, he has to decide whether to invest a positive capital amount in the modern sector. So the entrepreneur will compare his payoff whether $k_M = 0$ with his payoff whether $k_M > 0$. He will invest no capital amount in the modern sector if and only if

$$\pi^{(2)}_1 > \pi^{(2)}_4 \Rightarrow 2(k_T+k_M)^\beta \geq -2sk_M + k^\alpha_M + k^\beta_T + (k_T + k_M)^\beta - q(m + c)k_M$$

(see (2) and (5)) that is

$$-2sk_M + k^\alpha_M + k^\beta_T - (k_T + k_M)^\beta - q(m + c)k_M \leq 0.$$

It is verified if and only if

$$q \geq \frac{k^\alpha_M + k^\beta_T - (k_T + k_M)^\beta - 2sk_M}{(m + c)k_M} = q_2.$$

If this inequality holds, the entrepreneur will invest all his capital in the traditional sector. Otherwise, assume $q_2 > 0$, then if $q < q_2$ the entrepreneur will invest a positive quantity in the modern sector.\(^{15}\) It is straightforward to prove that $q_2 < q_1$.

\(^{15}\)If $q_2 \leq 0$ the entrepreneur will never invest in the modern sector.
C Appendix

Proof of Proposition 3.1.

Proof. First we prove part (a). If $q \in [0, q_2]$ then the payoff vector is given by (7) as we have proven in proposition 2.2 so the optimal capital amount $k_M$ to be invested in the modern sector is given by the solution to the following maximum problem faced by the entrepreneur

$$
\max_{k_M \in (0,1]} \frac{k_M^\alpha + k_T^\beta + (k_T + k_M)^\beta - q(m + c)k_M - 2sk_M}{2}
$$

subject to the restriction $k_T = 1 - k_M$. We substitute the boundary in the objective function and we maximize

$$
f(k_M) = \frac{k_M^\alpha + (1 - k_M)^\beta - q(m + c)k_M - 2sk_M}{2}
$$

The necessary condition is

$$
f'(k_M) = \frac{\alpha k_M^{\alpha-1} - \beta (1 - k_M)^{\beta-1} - q(m + c) - 2s}{2} = 0 \Rightarrow
\alpha k_M^{\alpha-1} - \beta (1 - k_M)^{\beta-1} = q(m + c) + 2s.
$$

Let $\gamma_1 = q(m + c) + 2s > 0$ and $g(k_M) = \alpha k_M^{\alpha-1} - \beta (1 - k_M)^{\beta-1}$. Then $g(k_M)$ is a continuous strictly decreasing function such that $\lim_{k_M \to 0^+} g(k_M) = +\infty$ while $\lim_{k_M \to 1^-} g(k_M) = -\infty$ so that equation $g(k_M) = \gamma_1$ has a unique solution $k_M^* \forall \gamma_1$ such that $k_M^*$ is a decreasing function of $q$ that is $k_M^* = \phi(q)$ and $\phi'(q) < 0$.

Furthermore $k_M^*$ is a maximum point since $f$ is concave.

If $q \in (q_2, q_1)$ then the payoff vector is given by (2) and all the capital is invested in the traditional sector so part (b) is proven.

To prove part (c) we have to solve the following maximum problem

$$
\max_{k_M \in (0,1]} k_M^\alpha + k_T^\beta - sk_M
$$

subject to the restriction

$$
k_T = 1 - k_M
$$
Let $h(k_M) = k_M^\alpha + (1 - k_M)^\beta - sk_M$ then the first order condition, that is also sufficient $h$ being concave, is given by

$$h'(k_M) = \alpha k_M^{\alpha - 1} - \beta (1 - k_M)^{\beta - 1} - s = 0 \Rightarrow$$

$$\alpha k_M^{\alpha - 1} - \beta (1 - k_M)^{\beta - 1} = s.$$  

Let $\gamma_2 = s > 0$ and $g(k_M) = \alpha k_M^{\alpha - 1} - \beta (1 - k_M)^{\beta - 1}$ then following the same arguments we have used to prove part (a) it is proven that a unique value $k_M^* \in (0, 1)$ does exist such that the payoff for the entrepreneur is maximum, $\forall \gamma_2$, and it does not depend on $q$.

\[\square\]

### D Appendix

Proof of Proposition 3.2.

**Proof.** We first consider the properties of the production function. Let us consider the production function in (1) so being $k_M \in [0, 1]$, $y(k_M) = k_M^\alpha + (1 - k_M)^\beta$ is such that $y(0) = y(1) = 1$ and $y(k_M) > 1, \forall k_M \in (0, 1)$. Furthermore $y'(k_M) = \alpha k_M^{\alpha - 1} - \beta (1 - k_M)^{\beta - 1} = g(k_M)$ (see figure 1 (a)) so that $y$ has a unique maximum point $\bar{k}_M$ such that $g(\bar{k}_M) = 0$. Now we consider the three different cases.

Case (1). Let $q \in [0, q_2]$. Then the equilibrium capital amount invested in the modern sector is $k_M^*$ such that $g(k_M^*) = \gamma_1$ being $\gamma_1 > 0$ so $k_M^*$ belongs to the increasing side of $y(k_M)$ $\forall q \in [0, q_2]$. Furthermore $k_M^* = \phi(q)$ is a decreasing function of $q$ then $y(k_M^*) = y(\phi(q))$ is decreasing $\forall q \in [0, q_2]$. Notice that the equilibrium amount of capital invested in the modern sector without corruption, given by $g(k_M^*) = 0$, is bigger than the one in the case of corruption.

Case (2). Let $q \in (q_2, q_1)$. Then $k_M = 0$ and the production equilibrium level is $y(0) = 1$.

Case (3). Let $q \in [q_1, 1]$. Then the production equilibrium level is $\varphi = y(\kappa)$ such that $\varphi > y(k_M')$ since $\kappa > k_M'$ and also $g(\kappa) > 0$.

Function $y(q) = y(\psi(q))$ is upper–hemicontinuous since it is the composition between a continuous function and an upper–hemicontinuous function. 

\[\square\]
References


