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Abstract

In this paper we propose new panel tests to detect changes in persistence. The test statistics are used to test the null hypothesis of stationarity against the alternative of a change in persistence from $I(0)$ to $I(1)$, from $I(1)$ to $I(0)$, and in an unknown direction. The limiting distributions of the tests under the hypothesis of cross-sectional independence are derived. Cross-sectional dependence is also considered. The tests are applied to the inflation rates of 19 OECD countries over the period 1972-2008. Evidence of a change in persistence from $I(1)$ to $I(0)$ is found for a set of these countries.

Keywords: Persistence, Stationarity, Panel data.

JEL Classification: C12, C23.

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1 Introduction

There is a considerable literature investigating persistence in inflation which is generally measured by means of unit root tests. If the null hypothesis cannot be rejected, any shock to inflation has a permanent effect, and conversely, if the null can be rejected this effect is transitory. Whether inflation series should be considered as stationary or non-stationary has not yet been conclusively resolved (see Clark, 2006; Costantini and Lupi, 2007).

However, economic time series such as inflation rate may be characterized by a change in persistence between separate I(1) and I(0) regimes rather than simply I(1) or I(0) behavior. Barsky (1987) finds that inflation in the US evolved from being essentially a white noise process in the pre-World War I years to a highly persistent, nonstationary ARIMA process in the post-1960 period. Alogoskoufis and Smith (1991) argue that the degree of persistence of US inflation has changed significantly over time, becoming highly persistent in the post World War II era with respect to the level recorded during the US gold standard. Emery (1994) argues that persistence in US inflation decreases substantially throughout 1980s. According to Emery, US inflation can best be described as white noise. Using US quarterly inflation rate data over the period 1948:2-1994:1, Kim (2000) finds evidence in favor of a change from stationarity to a unit root around 1973:3 by applying a new residual-based ratio test for a change in persistence from stationarity to difference stationarity. Busetti and Taylor (2004) propose new ratio-based tests and breakpoint estimators which are consistent under I(1) to I(0) changes, and demonstrate that the ratio-based tests which are consistent against changes from I(1) to I(0) are not consistent against changes from I(0) to I(1), and viceversa. By applying these new tests to the US inflation rate, Busetti and Taylor provide evidence in favor of a shift from I(1) to I(0) with the estimated breaks located at the time of the US recession of 1990-1991. Harvey et al. (2006) find evidence for a shift from I(1) to I(0) using a modified version of the ratio-based statistics of Kim (2000), Kim et al. (2002) and Busetti and Taylor (2004).
In this paper we propose a set of new panel tests to detect changes in persistence based on the previous time series approach of Kim (2000), Kim et al. (2002), Busetti and Taylor (2004) and Harvey et al. (2006). The panel tests are used to test the null hypothesis of stationarity against the alternative of a change in persistence from I(0) to I(1), from I(1) to I(0) and when the direction is unknown. Since under the null hypothesis these tests assume no change in persistence for all cross-units, the null hypothesis may be rejected even if only one of the cross-section units show a change in persistence. Thus the panel tests are sensitive to the selection of the units in the panel. Following Chortareas and Kapetanios (2009), a procedure to identify the series which undergo changes in persistence is provided.\(^1\) Two sets of panel tests are proposed here. The first set is assigned to cross-sectionally independent panels. The asymptotic distributions of these tests are derived and are shown to be normally distributed. The second set uses the hypothesis of cross-section dependence, and defactored data are then applied. The sample size and power of the tests are then investigated in a Monte Carlo experiment.

The paper is organized as follows. Section 2 presents the new panel tests under the hypothesis of cross-section independence. Section 3 describes the panel tests under the cross-section dependence. Section 4 presents the Monte Carlo simulations. In section 5 the new panel tests are applied to 19 OECD quarterly inflation rates over the period 1972:2-2008:2. Section 6 concludes. The main technical proofs and derivations can be found in the Appendix.

\(^1\)In a recent paper, Costantini and Gutierrez (2007) propose panel tests for a change in persistence, although they do not provide a procedure to detect the direction of the change in persistence nor a method to obtain estimates of the break dates.
2 Persistence tests without cross-section correlation

2.1 The model

In this section we focus on the following Gaussian unobserved components model:

\[ y_{i,t} = d_i + \mu_{i,t} + \varepsilon_{i,t}, \quad i = 1, \ldots, N, \; t = 1, \ldots, T, \]  

(1)

considering three possible cases of changes in persistence:

- **Case 1:** \( I(0) \rightarrow I(1) \)
  
  \[ \mu_{i,t} = \mu_{i,t-1} + 1(t > [T \tau_i]) \eta_{i,t}, \quad i = 1, \ldots, N, \; t = 1, \ldots, T, \]  

(2)

- **Case 2:** \( I(1) \rightarrow I(0) \)
  
  \[ \mu_{i,t} = \mu_{i,t-1} + 1(t \leq [\tau_i T]) \eta_{i,t} \quad i = 1, \ldots, N, \; t = 1, \ldots, T. \]  

(3)

- **Case 3:** unknown direction \( I(0) \rightarrow I(1) \) or \( I(1) \rightarrow I(0) \)

where \( 1(\cdot) \) is the indicator function, \( d_i \) is a deterministic component, \( \varepsilon_{i,t} \) and \( \eta_{i,t} \) are mutually independent mean zero i.i.d. gaussian processes with \( \sigma^2_\varepsilon \) and \( \sigma^2_\eta \) variances respectively.

As regards equation (2), for each cross section \( i \) the data generating process yields a process which is stationary up to and including time \([\tau_i T]\), with a change-point proportion \( \tau_i \in (0,1) \), but it is non-stationary after the break, if and only if \( \sigma^2_\eta > 0 \). With respect to equation (3), the data generating process yields a process which is non-stationary up to and including time \([\tau_i T]\) but it is stationary after the break, if and only if \( \sigma^2_\eta > 0 \).

Therefore, the panel test of stationarity against a shift in persistence from stationarity to a unit root or viceversa involves testing the null hypothesis as given in:

\[ H_0 : \sigma^2_{\eta i} = 0, \; \forall i \]  

(4)

against the alternative hypothesis

\[ H_1 : \sigma^2_{\eta i} > 0, \; \text{at least for some } i. \]  

(5)
When $I(0) \rightarrow I(1)$, the alternative hypothesis is denoted as $H_{01}$. If $I(1) \rightarrow I(0)$, then $H_{10}$ is used.

The following assumption plays a key role throughout the paper.

**Assumption 1** Fixed $i \in N$, the process $\{\mu_{i,t}\}_{t=0}^{\infty}$ is such that

1. $E[\mu_i] = 0$;

2. $E|\mu_i|^4 < +\infty$;

3. $\{\mu_{i,t}\}_{t=0}^{\infty}$ is $\phi$-mixing with mixing coefficients $\phi_{i,m}$ such that

$$\sum_{m=1}^{\infty} \phi_{i,m}^{\gamma_i} < +\infty,$$

for some $\gamma_i > 0$;

4. there exists the long-run variance

$$\sigma_{\mu_i}^2 = \sum_{j=0}^{\infty} E[\mu_{i,j+1}\mu_{i,1}];$$

5. for each $s \in (0, 1)$, we have

$$\lim_{T \to \infty} V\left[\frac{1}{\sqrt{T}} \sum_{t=sT+1}^{[sT]} \mu_{i,t}\right] = s\sigma_{\mu_i}^2$$

and

$$\lim_{T \to \infty} V\left[\frac{1}{\sqrt{T}} \sum_{t=sT+1}^{T} \mu_{i,t}\right] = (1-s)\sigma_{\mu_i}^2.$$

The above conditions are used by Phillips (1987), Phillips and Perron (1988) and Phillips and Solo (1992), among others, to obtain limiting distribution of a stochastic process. Throughout the following sections sequential limits, where $T \to \infty$ is followed by $N \to \infty$, are used.

### 2.2 Panel ratio-based tests: $I(0) \rightarrow I(1)$

In this section we present new panel tests to detect changes in persistence according to equation (2) and investigate their asymptotic behavior. The panel tests are shown to be standard normally
The gaussian process (1)-(2) is considered in order to test the null hypothesis $H_0$ against $H_{01}$. Let $\tilde{\epsilon}_{i,t}, i = 1, \ldots, N$ and $t = 1, \ldots, T$, be the residuals from the regression of $y_{i,t}$ on intercept. If a structural change occurs at time $t = [\tau_i T]$ for $\tau_i \in (0, 1)$, the following partial sum process can be defined as:

$$
\begin{align*}
& S_{i,t}^{(0)} = \sum_{j=1}^{t} \tilde{\epsilon}_{i,j}, \quad t = 1, \ldots, [T \tau_i]; \quad i = 1, \ldots, N, \\
& S_{i,t}^{(1)} = \sum_{j=[T \tau_i] + 1}^{t} \tilde{\epsilon}_{i,j}, \quad t = [T \tau_i] + 1, \ldots, T; \quad i = 1, \ldots, N,
\end{align*}
$$

Then the following time series test is considered:

$$
\tilde{K}_{T,i}(\tau_i) = \frac{(T - [T \tau_i])^{-2} \sum_{t=[T \tau_i] + 1}^{T} S_{i,t}^{(1)}(\tau_i)^2}{\sum_{t=1}^{[T \tau_i]} S_{i,t}^{(0)}(\tau_i)^2}, \quad i = 1, \ldots, N.
$$

(7)

Since the true value of $\tau_i$ in (7) is unknown, three transformations of the tests $\tilde{K}_{T,i}(\tau)$ for testing changes in persistence are considered in a time series framework and then extended to a panel setting. These three time series transformations are:


$$
H_1(\tilde{K}_{T,i}(\tau_i)) := \sup_{\tau_i \in (0,1)} \tilde{K}_{T,i}(\tau_i), \quad i = 1, \ldots, N.
$$

(8)

- The mean-exponential test introduced by Andrews and Ploberger (1994)

$$
H_2(\tilde{K}_{T,i}(\tau_i)) := \log \left\{ \int_{\tau_i \in (0,1)} \exp[\tilde{K}_{T,i}(\tau_i)] d\tau_i \right\}, \quad i = 1, \ldots, N.
$$

(9)

- The mean score test proposed by Hansen (1991)

$$
H_3(\tilde{K}_{T,i}(\tau_i)) := \int_{\tau_i \in (0,1)} \tilde{K}_{T,i}(\tau_i) d\tau_i, \quad i = 1, \ldots, N.
$$

(10)

On the basis of equations (8), (9) and (10), we now construct three panel tests:

$$
Q_j^{(T,N)} := \frac{1}{\sigma_j^{(Q)}} \sqrt{N} \sum_{i=1}^{N} \left[ H_j(\tilde{K}_{T,i}(\tau_i)) - \mu_j^{(Q)} \right], \quad \text{for } j = 1, 2, 3,
$$

(11)
where
\[ \mu_j^{(Q)}(Q) = E\left[H_j(\tilde{K}_{T,i}(\tau_i))\right], \quad i = 1, \ldots, N; \quad (12) \]
and
\[ \sigma_j^{(Q)}(Q) = \sqrt{V\left[H_j(\tilde{K}_{T,i}(\tau_i))\right]}, \quad i = 1, \ldots, N. \quad (13) \]

The mean and the variance (12) and (13) are computed using the i.i.d. hypothesis for all \( \tilde{\varepsilon} \) and \( \tau \).

The asymptotic distributions of the tests in (11) are contained in the following theorem.

**Theorem 2.1** It results
\[ \lim_{N \to +\infty} \lim_{T \to +\infty} Q_{T,N}^{(T,N)} \sim N(0, 1), \quad j = 1, 2, 3. \]

### 2.3 Panel reverse test: I(1) \( \to \) I(0)

Let us consider the gaussian process (1)-(3) in which the null hypothesis refers to the stationary process and the alternative to a shift from I(1) to I(0). The following reverse test statistic is introduced:
\[ \tilde{K}_{T,i}(\tau_i) = (\tilde{K}_{T,i}(\tau_i))^{-1}, \quad i = 1, \ldots, N. \quad (14) \]

where the true value of \( \tau_i \) is unknown as in equation (7).

Applying the procedure described in subsection 2.2 to (14), three reverse panel tests are developed:
\[ R_{T,N}^{(R)} := \frac{1}{\sigma_j^{(R)}\sqrt{N}} \sum_{i=1}^{N} \left[H_j(\tilde{K}_{T,i}(\tau_i)) - \mu_j^{(R)}\right], \quad \text{for } j = 1, 2, 3, \quad (15) \]

where
\[ \mu_j^{(R)} = E\left[H_j(\tilde{K}_{T,i}(\tau_i))\right], \quad i = 1, \ldots, N; \quad (16) \]
and
\[ \sigma_j^{(R)} = \sqrt{V\left[H_j(\tilde{K}_{T,i}(\tau_i))\right]}, \quad i = 1, \ldots, N. \quad (17) \]

The following result shows the asymptotic distributions of the tests in (15).
Theorem 2.2 It results

\[ \lim_{N \to +\infty} \lim_{T \to +\infty} R_j^{(T,N)} \sim N(0,1), \quad j = 1, 2, 3. \]

2.4 Panel tests with an unknown direction

As in the previous sections, three panel tests are developed and their asymptotic distributions are derived. The panel tests are:

\[ M_j^{(T,N)} = \frac{1}{\sigma_j^{(M)} \sqrt{N}} \sum_{i=1}^{N} [\max\{H_j(\tilde{K}_{T,i}), H_j(\tilde{K}_{T,i}^*)\} - \mu_j^{(M)}], \quad j = 1, 2, 3; \quad (18) \]

where

\[ \mu_j^{(M)} = \mathbb{E} \left[ \max\{H_j(\tilde{K}_{T,i}), H_j(\tilde{K}_{T,i}^*)\} \right] \quad j = 1, 2, 3; \quad i = 1, \ldots, N; \quad (19) \]

\[ \sigma_j^{(M)} = \sqrt{\mathbb{V} \left[ \max\{H_j(\tilde{K}_{T,i}), H_j(\tilde{K}_{T,i}^*)\} \right]} \quad j = 1, 2, 3; \quad i = 1, \ldots, N. \quad (20) \]

The asymptotic distributions of these tests are now derived.

Theorem 2.3 It results

\[ \lim_{N \to +\infty} \lim_{T \to +\infty} M_j^{(T,N)} \sim N(0,1). \]

2.5 Modified panel tests

An important issue regarding the test procedures in the previous sections is the behavior of the statistics in the absence of a change in persistence, yet the null hypothesis is not true. That is, for a test with a null of \( I(0) \) throughout, what happens when a series is actually \( I(1) \) throughout? Harvey et al. (2006) show that Kim (2000) and Busetti and Taylor (2004) tests are severely oversized in this case, demonstrating frequent spurious rejections of the \( I(0) \) null in favor of a change in persistence. In order to deal with this issue, we propose a modified panel versions of the statistics developed in subsections 2.2-2.4 using the same approach of Harvey et al. (2006). These modified tests have the same critical values in the limit as the corresponding unmodified tests under the \( \text{H}_0 \), and also under \( \text{H}_1 \), since the modification has no asymptotic effect under the null, and under
the alternative the modification is taken such that the limiting critical value is precisely the same under $H_0$. The modified panel tests are:

$$MQ_j^{(T,N)} := \exp(-bJ_{1,N,T}) \cdot Q_j^{(T,N)}, \quad j = 1, 2, 3; \quad (21)$$

$$MR_j^{(T,N)} := \exp(-bJ_{1,N,T}) \cdot R_j^{(T,N)}, \quad j = 1, 2, 3; \quad (22)$$

$$MM_j^{(T,N)} := \exp(-bJ_{1,N,T}) \cdot M_j^{(T,N)}, \quad j = 1, 2, 3; \quad (23)$$

where $b$ is a finite constant and $J_{1,N,T}$ is the arithmetic mean of $N$ of the truncated sequences of $T^{-1}$ times the Wald statistic $J_{1,T}^{(i)}$ to test the joint hypothesis $\varsigma_{i,1} = \cdots = \varsigma_{i,9} = 0$ in a panel regression

$$y_{i,t} = \epsilon_{i,t} + \sum_{j=1}^9 \varsigma_{i,j}t^j + \text{error}, \quad t = 1, \ldots, [\tau_t]; \quad i = 1, \ldots, N. \quad (24)$$

Under the null hypothesis, Harvey et al. (2006) show that

$$\lim_{T \rightarrow +\infty} J_{1,T}^{(i)} = 1, \quad i = 1, \ldots, N.$$

Since an i.i.d. hypothesis is assumed with respect to the cross-sectional dimension $i$, we have

$$\lim_{T,N \rightarrow +\infty} J_{1,N,T} = \lim_{T \rightarrow +\infty} \frac{1}{N} \sum_{i=1}^N J_{1,T}^{(i)} = 1.$$ 

Consequently the modified panel tests have the same limiting distribution under $H_0$ as the unmodified tests. Under the alternative hypothesis, using Harvey et al. (2006) results and the fact that the asymptotic distributions of tests $Q_j^{(T,N)}$, $R_j^{(T,N)}$, and $M_j^{(T,N)}$, $j = 1, 2, 3$ are standard gaussian (see Theorem 2.1, Theorem 2.2 and Theorem 2.3), we have

$$\lim_{N \rightarrow +\infty} T^{-2}(MQ_j^{(T,N)} - Q_j^{(T,N)}) =$$

$$= T^{-2} \lim_{N \rightarrow +\infty} (MQ_j^{(T,N)} - Q_j^{(T,N)}) =$$

$$= T^{-2} \lim_{N \rightarrow +\infty} \{\exp[-bJ_{1,N,T}] - 1\} Q_j^{(T,N)} = O_p(1)O_p(1) = O_p(1).$$

Analogously, it results

$$\lim_{N \rightarrow +\infty} T^{-2}(MR_j^{(T,N)} - R_j^{(T,N)}) = O_p(1)$$
and
\[
\lim_{N \to +\infty} T^{-2}(M \Phi_{j}^{(T,N)} - \Phi_{j}^{(T,N)}) = O_p(1).
\]

Thus we obtain a test which rejects the null for large values and retains the same rate of consistency under the alternative \(H_{01}\) as in the original unmodified tests \(Q\)’s, \(R\)’s and \(M\)’s. The modified tests are also \(O_p(1)\) under the alternative \(H_{10}\).

A further appropriate modification procedure is proposed to test the null against the alternative \(H_{01}\). Following Harvey et al. (2006), tests (21), (22) and (23) are modified by introducing

\[
J_{\min,N,T} := \frac{1}{N} \sum_{i=1}^{N} \left[ \min_{\tau_i \in (0,1)} J_{i,T}^{(j)} \right].
\]

We define

\[
M \Phi_{min,N}^{(T)} := \exp(-bJ_{min,N,T}) \cdot \Phi_{j}^{(T,N)}, \quad j = 1, 2, 3; \quad (25)
\]

\[
M \mathcal{R}_{min,N}^{(T)} := \exp(-bJ_{min,N,T}) \cdot \mathcal{R}_{j}^{(T,N)}, \quad j = 1, 2, 3; \quad (26)
\]

\[
M \mathcal{M}_{min,N}^{(T)} := \exp(-bJ_{min,N,T}) \cdot \mathcal{M}_{j}^{(T,N)}, \quad j = 1, 2, 3; \quad (27)
\]

and

\[
J_{\min,T} := \lim_{N \to +\infty} J_{\min,N,T}.
\]

The asymptotic distributions of the tests are derived using the alternative hypothesis of Harvey et al. (2006). Thus, we have

\[
\lim_{N \to +\infty} T^{-2}(M \Phi_{min,N}^{(T)} - \Phi_{j}^{(T,N)}) =
\]

\[
= T^{-2} \lim_{N \to +\infty} (M \Phi_{min,N}^{(T)} - \Phi_{j}^{(T,N)}) =
\]

\[
= T^{-2} \left\{ \exp[-bJ_{min,N,T}] - 1 \right\} Q_{j}^{(T,N)} =
\]

\[
= T^{-2} \left\{ \exp[-bJ_{min,N,T}] - 1 \right\} \lim_{N \to +\infty} Q_{j}^{(T,N)} = \alpha_p(1) = O_p(1),
\]

and, analogously,

\[
\lim_{N \to +\infty} T^{-2}(M \mathcal{R}_{min,N}^{(T)} - \mathcal{R}_{j}^{(T,N)}) = O_p(1)
\]
and

\[ \lim_{N \to +\infty} T^{-2}(M_{\text{min},j}^{(T,N)} - M_j^{(T,N)}) = o_p(1). \]

Thus the above modification has no asymptotic effect under the alternative \( H_{01} \), unlike the original modifications \( M_Q \)'s, \( M_R \)'s and \( M_M \)'s in (21), (22) and (23). It is easily shown that the modified min tests are \( O_p(1) \) under the alternative \( H_{10} \).

3 Persistence test with cross-section correlation

Whereas the previous testing procedures are valid under the assumption that the units are cross-section independent, this assumption is restrictive in many empirical applications since countries or regions are cross-correlated. Thus cross-section dependence across units is considered using a procedure of filtering out common factors in the panel structure (for a survey on cross-section correction methods see Breitung and Pesaran, 2008). The estimation procedure is based on the approaches of Stock and Watson (2002), Bai (2003) and Bai and Ng (2004), and it consists of two steps. In the first step, Principal Component (PC) method is applied to estimate the true number of common factors, which are computed using Bai and Ng’s (2002) selection criteria. In the second step, defactored data are then used.

3.1 The model

In this section we focus on the following model:

\[ y_{i,t} = d_i + \mu_{i,t} + \epsilon_{i,t}, \quad i = 1, \ldots, N, t = 1, \ldots, T, \]  
(28)

considering three cases of changes in persistence as in the previous section:

- Case 1: \( I(0) \to I(1) \)

\[ \mu_{i,t} = \mu_{i,t-1} + 1(t > [\tau_i T])\eta_{i,t} \]  
(29)

\[ \epsilon_{i,t} = F_{i}^{\lambda_i} + u_{i,t} \]  
(30)
Case 2: $I(1) \rightarrow I(0)$

\[
\mu_{i,t} = \mu_{i,t-1} + 1(t \leq \lceil \tau_i T \rceil)\eta_{i,t}
\]

(31)

\[
\varepsilon_{i,t} = F_t \lambda_i + u_{i,t}
\]

(32)

Case 3: unknown direction $I(0) \rightarrow I(1)$ or $I(1) \rightarrow I(0)$

where $u_{i,t}$ is a stationary process, $\tau_i \in (0,1)$, and $d_i$ is the deterministic component. Furthermore, we assume that $F_t$ in (28) is a stationary $(r \times 1)$ vector of the common factors and $\lambda_i$ is the corresponding vector of the factor loadings. The factors $F_t$ and factor loadings $\lambda_i$ in (28) can be written in matrix notation as follows: $F = (F_1, \ldots, F_T)'$ and $\Lambda = (\lambda_1, \ldots, \lambda_N)'$.

Let $M < \infty$ be a positive number which does not depend on $T$ or $N$. Let $\|A\| = \text{trace}(A'A)^{1/2}$. Our analysis is based on the following assumptions:

**Assumption 2** The loadings $\lambda_i$ is either deterministic such that $\|\lambda_i\|^4 \leq M$ or stochastic such that $E\|\lambda_i\|^4 \leq M$, in either case $\Lambda_i'\Lambda_i/N \rightarrow \Sigma_\Lambda$, as $N \rightarrow \infty$ for some $(r \times r)$ positive definite matrix $\Sigma_\Lambda$.

**Assumption 3** $E\|F_t\|^4 \leq M$ and $\frac{1}{T} \sum_{t=1}^{T} F_tF_t' \rightarrow \Sigma_F$, for a $(r \times r)$ positive definite matrix $\Sigma_F$.

**Assumption 4** The errors $\{\varepsilon_{i,t}\}$ and $\{\eta_{i,t}\}$, the factor $\{F_t\}$ and the loadings $\{\lambda_i\}$ are four mutually independent stochastic variables.

Assumption 2 is made on the factor loadings in order to ensure that the factor structure can be identified. Assumption 3 is more general than the one in classical factor analysis in which the factors are i.i.d. and they are allowed to be dynamic.

The common factors are estimated using the approach of Stock and Watson (2002), where the principal component of $F$, denoted as $\hat{F}$, is $\sqrt{T}$ times the first $r$ eigenvectors, corresponding to the first $r$ largest eigenvalues of the $(T \times T)$ matrix of demeaned and standardized $\hat{y}_t \hat{y}_t'$. Under
the normalization $\hat{F}^\prime / T = I_r$, the estimated loading matrix is $\hat{\Lambda} = \hat{F}^\prime \hat{y}_i / T$. Thus the estimated residuals are defined as

$$\hat{z}_{i,t} = \hat{y}_{i,t} - \hat{F}_t \hat{\lambda}_i$$  \hspace{1cm} (33)$$

According to data generating process (28)-(29) and (33), one can see that for each cross section $i$, the process $\hat{z}_{i,t}$ is stationary up to and including time $[\tau, T]$ but it is nonstationary after the break, if and only if $\sigma_{\eta_i}^2 > 0$. Naturally the converse is true if (32) is used instead of (29).
Thus our strategy is to apply the panel test statistics presented in section 2 to the de-factored data $\hat{z}_{i,t}$.

3.2 Consistency of $F_t$

In this section it is shown that $F_t$ may be consistently estimated in the presence of a break.\(^2\) To fix ideas, the case of $I(0) \rightarrow I(1)$ is considered. Model (28) may be described as follows:

$$y_{i,t} = d_i + \mu_{i,t} + F_t \lambda_i + u_{i,t}$$  \hspace{1cm} (34)$$

Considering the first difference of model (34), we have

$$\Delta y_{i,t} = \Delta \mu_{i,t} + \Delta F_t \lambda_i + \Delta u_{i,t}$$  \hspace{1cm} (35)$$

In the presence of a break, $\Delta \mu_{i,t}$ may result as being either stationary or nonstationary process as shown below:

I. $\Delta \mu_{i,t}$ is stationary. In this case the Bai and Ng (2002) results may be used in order to obtain the consistency of the estimated factors.

II. $\Delta \mu_{i,t}$ is nonstationary. In this case the results of Bai and Ng (2002) cannot be used since the stationarity of the process $\Delta y_{i,t}$ is not known a priori.

\(^2\)In the case of the absence of a break, the consistency of estimated factor is obtained in Bai and Ng (2004). This procedure can not apply in our case.
Let us check if $\Delta \mu_{i,t}$ is stationary or not.

**Theorem 3.1** Assume that $\tau_i$ is a uniform random variable. Then $\Delta \mu_{i,t}$ is a stationary process.

**Remark 5** The assumption regarding the distribution of $\tau_i$ in Theorem 3.1 is reasonable. Indeed, a break may occur at any time in the sample period.

Using theorem 3.1 the procedure of Bai and Ng (2002, 2004) can be used since the process is stationary as in I.

### 4 Selecting persistence across units in the panel

Following the approach of Chortareas and Kapetanios (2009), this section describes a procedure which separates the set of the series into a group of series with a break and a group of those without.

For each series $y_{i,t}$, $i = 1, \ldots, N$ and $t > 0$, a binary variable is defined

$$G_i := \begin{cases} 
0, & \sigma^2_{\eta_i} > 0; \\
1, & \sigma^2_{\eta_i} = 0. 
\end{cases}$$  \hspace{1cm} (36)

$G$’s are collecting by defining a set of indexes $i_M := \{i_1, \ldots, i_M\}$, with $M \leq N$, and the vector $G_{i_M} := (G_{i_1}, \ldots, G_{i_M})$. The time series vector $y_{i_M,t} := (y_{i_1,t}, \ldots, y_{i_M,t})$ is also introduced. We wish to estimate the componentwise of $G_{i_M}$. The estimates are denoted by $\hat{G}_{i_M} = (\hat{G}_{i_1}, \ldots, \hat{G}_{i_M})$.

In order to distinguish a series with a break from one without, the panel test $\Xi$ and modifications of $\Xi$ (see section 5 for the application) may be used in place of $Q_j^{(T,N)}$, $\mathcal{R}_j^{(T,N)}$ and $\mathcal{M}_j^{(T,N)}$ defined above. Analogously, the time series statistics $H_j(\tilde{K}_{T,j})$, $H_j(\tilde{K}_{T,j})$ and $\max\{H_j(\tilde{K}_{T,j}),H_j(\tilde{K}_{T,j})\}$ are denoted $\xi_i$. The statistics $\Xi$ and $\xi_i$ are *conjugated* if and only if $\Xi$ is the panel statistics constructed from the time series test $\xi_i$. In order to estimate $G_{i_M}$, the following algorithm is proposed:

1. Set $k = 1$ and $i^{(k)} := \{1, \ldots, N\}$. 

2 Calculate the panel statistic $\Xi$ for the set of series $y_{i(k),t}$. If the test does not reject the null hypothesis, stop and set $\hat{G}_{i(k)} = (0, \ldots, 0)$. If the null hypothesis is rejected, go on to the next step.

3 Set $\hat{G}_s = 1$, where $s$ is the index of the series associated with the maximum value of $\xi_i$ which is conjugated with $\Xi$. Set $k = k + 1$ and $i^{(k+1)} := \{1, \ldots, s - 1, s + 1, \ldots, N\}$. Go to step 2.

The consistency of $\hat{G}_{1M}$ as an estimator of $G_{1M}$ is defined.

**Theorem 4.1** Let $\alpha_T$ be the significance level used for the panel test $\Xi$. Assume that:

(i) $\lim_{T \to +\infty} \alpha_T = 0$;

(ii) $\lim_{T \to +\infty} \frac{\log(\alpha_T)}{T^2 \sqrt{N}} = 0$.

Then

$$\lim_{T \to +\infty} P \left( \sum_{s=1}^{N} |\hat{G}_s - G_s| > 0 \right) = 0.$$  \hfill (37)

Let us denote the number of the series with a break as $N_2$, where $N_1 := N - N_2$. The set of indexes related to the series with a break is denoted as $\mathcal{I}_b$, and $|\mathcal{I}_b| = N_2$.

**Theorem 4.2** If $N, T \to +\infty$, then:

(i) $G_s = 1$ implies that

$$\lim_{T \to +\infty} P(\hat{G}_s = 1) = 1;$$  \hfill (38)

(ii) $G_s = 0$ implies the existence of a constant $\lambda$ such that

$$\lim_{N_2 \to +\infty} \lim_{T \to +\infty} P \left( \sum_{s \in \mathcal{I}_b} |\hat{G}_s - G_s| > \lambda \right) = 0.$$  \hfill (39)

### 4.1 Estimation of the break

In this subsection, a break estimation procedure is proposed for a change in persistence from $I(0)$ to $I(1)$ and vice versa. Without loss of generality, the time series can be ordinated and it may be
assumed that $I_b = \{1, \ldots, N_2\}$. In order to obtain the asymptotic behavior of the break estimators, the following assumption is made.

**Assumption 6** Let $\tilde{\mu}_{i,s+1}, \tilde{\mu}_{i,s+2}, \ldots, \tilde{\mu}_{i,s+m}$, for $s \in 0, \ldots, T - 1$ and $m \leq T - s$ be a sequence of stationary variables. Assume that $m^{-1} \sum_{t=s+1}^{s+m} \tilde{\mu}_{i,t}^2 \to E[\mu_i^2]$ for $E[\mu_i^2] < \infty$, $\forall i = 1, \ldots, N_2$.

- Case 1: $I(0) \to I(1)$.

Consider the following time series estimator:

$$
\pi_{i,T}^{(0-1)}(\tau_i) = \frac{\sum_{t=[T\tau_i]+1}^{T} \tilde{\mu}_{i,t}^2 / (T - [T\tau_i])^2}{\sum_{t=1}^{[T\tau_i]} \tilde{\mu}_{i,t}^2 / [T\tau_i]}, \quad i = 1, \ldots, N_2.
$$

(40)

Now, let $\hat{\tau}_i^{(0-1)}$ be:

$$
\hat{\tau}_i^{(0-1)} = \{\text{argmax}_{\tau_i \in (0,1)} \pi_{i,T}^{(0-1)}(\tau_i)\}, \quad i = 1, \ldots, N_2.
$$

(41)

Formula (41) provides a time series estimate of the unknown change point.

- Case 2: $I(1) \to I(0)$.

The time series estimator is now defined as follows:

$$
\pi_{i,T}^{(1-0)}(\tau_i) = \frac{\sum_{t=[T\tau_i]}^{[T\tau_i]+1} \tilde{\mu}_{i,t}^2 / (T - [T\tau_i])^2}{\sum_{t=[T\tau_i]+1}^{T} \tilde{\mu}_{i,t}^2 / (T - [T\tau_i])}, \quad i = 1, \ldots, N_2.
$$

(42)

With respect to the estimate of the unknown change point, we have:

$$
\hat{\tau}_i^{(1-0)} = \{\text{argmax}_{\tau_i \in (0,1)} \pi_{i,T}^{(1-0)}(\tau_i)\}, \quad i = 1, \ldots, N_2.
$$

(43)

The following theorem is a direct consequence of Kim (2000, Theorem 3.5) and shows the asymptotic properties of the estimators $\hat{\tau}_i^{(0-1)}$ and $\hat{\tau}_i^{(1-0)}$.

**Theorem 7** Under the alternative hypothesis and under the Assumption 6, it results

$$
(\hat{\tau}_i^{(0-1)} - \tau_i) = (\hat{\tau}_i^{(1-0)} - \tau_i) = o_p(1),
$$

(44)

$$
T(\hat{\tau}_i^{(0-1)} - \tau_i) = T(\hat{\tau}_i^{(1-0)} - \tau_i) = O_p(1).
$$

(45)
5 Monte Carlo simulation results

In this section Monte Carlo experiments are used to investigate the finite sample properties of the panel persistence tests. Two sets of Monte Carlo experiments are considered. The first set focuses on the model (1)-(3), where cross-section independence is assumed, while the second set uses model (28)-(33) for dependent panels. We start the analysis by considering the empirical rejection frequencies of the tests when the data are generated according to a change from I(0) to I(1) as in model (1)-(3). The impact of varying the signal-to-noise ratio is investigated between $\sigma_{\eta i} = 0, 0.10, 0.50$ and $\sigma_{\epsilon i} \sim U[0.5, 1.5]$. The breakpoint is uniform distributed, $\tau \sim U[0.3, 0.7]$. Only the modified min panel tests are considered here for reasons of space (results for the other panel tests are available upon request from the authors). The simulation results are performed using 1000 Monte Carlo replications and the RNDN function of Gauss 6.0. For all the tests, we consider $N = 15, 25, 50, 100$ and $T = 50, 100$. To overcome ill-conditioning arising from regression (24), only a subset of $T$ can be used. Following the earlier literature, we focus the analysis on the subset $[\gamma T, (1 - \gamma)T]$ with $\gamma = 0.2$. This choice is applied throughout this and the next section of the paper. In Table 1 we present the moments of Kim’s (2000, 2002) and Busetti and Taylor’s (2004) tests which are used to standardize the panel tests. Their values are computed using 50,000 replications. In Tables 2-3 we report empirical rejection frequencies for the size ($H_0 : \sigma_{\eta i}^2 = 0$) and power ($H_1 : \sigma_{\eta i}^2 > 0$) of the panel tests for model (1)-(3). All the panel test statistics seem to have good size for both small and large $T, N$.

Many interesting results emerge regarding the power of the tests for case 1 where $I(0) \rightarrow I(1)$ (Table 2). The power of the tests grows as the signal-to-noise ratio rises, as is generally expected, since the higher $\sigma_\eta$ is, the stronger the random walk component. The opposite findings are found for panel reverse tests, since a change from I(0) to I(1) is being tested.

Simulation results for the case 2 (I(1) $\rightarrow$ I(0)) are reported in Table 3. The power of the tests

---

3 As regards the case of independent panel tests we also considered $N=1$. 

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17
grows largely for the reverse statistics and it is interesting to note that the results mimic those in Table 2. Finally it is also should be noted that the panel tests always have a higher power than the univariate tests. In order to take cross-section dependence into account we use the method proposed in Stock and Watson (2002) and Bai and Ng (2004). The method consists of filtering out the individual-specific cross-sections \( y_{it} \) by the factor component computed using the principal component method. We assume a single \( F_t \sim N(0, 1) \), and \( \Lambda \sim U[0, 1] \). The number of factors are computed using the approach of Bai and Ng (2002). Throughout the Monte Carlo simulation analyses the number of factors are computed using the \( IC(3) \) criterion proposed by Bai and Ng (2002) with a maximum number of three factors.

Tables 4-5 present the empirical rejection frequencies for the defactored panel tests, showing the empirical size \( (H_0 : \sigma^2_{\eta_i} = 0) \) and the power \( (H_1 : \sigma^2_{\eta_i} > 0) \). It is should be noted that the tests have generally good size. Table 4 power reports results for a change from I(1) to I(0). The power of the tests grows for larger values of \( T \) and \( N \), with the exception of the reverse panel tests since a change from I(0) to I(1) is being tested. Results of the power of tests for the case 2 (I(1) \( \rightarrow \) I(0)) are reported in Table 5. The power of the tests grows considerably for the reverse tests, as would be expected.

6 Empirical application

The new panel tests are used in the analysis of the inflation rates for a panel of 19 OECD countries.\(^4\) Quarterly CPI inflation is considered (measured as the annual change in the CPI) over the period 1972:2-2008:2. All data are taken from OECD Main Economic Indicators. Table 6 reports the application of the modified panel persistence change tests to the aforementioned data. With respect to the panel tests constructed under the hypothesis of cross section independence, the null

\(^4\)The selected countries are Australia, Austria, Belgium, Canada, Finland, France, Germany, Greece, Italy, Japan, Luxembourg, New Zealand, Norway, Portugal, Spain, Sweden, Switzerland, United Kingdom and United States.
hypothesis of stationarity cannot be rejected in the case of a shift from I(0) to I(1), while it is rejected for a change from I(1) to I(0) and also where the direction is unknown. These findings suggest a change of persistence from I(1) to I(0) for inflation rate. However, the previous results are valid under the assumption of cross-section independence which is rather unrealistic in empirical economic applications. Therefore, cross-section dependence is considered throughout the use of the factor model described in section 3.

Three factors are selected using the $IC(3)$ criterion (from a fixed maximum of five). The estimated factors and factor loadings are then used to obtain $\hat{z}_{i,t}$ as in equation (33) and the panel tests are applied to the defactored data. Results show a change in persistence from I(1) to I(0) as found in the case of cross-section independence.

Having rejected the null hypothesis of constant persistence, the countries with a break in the inflation rate must be identified. To this end the procedure presented in section 4 is used. The statistics provided by the procedure is 13.21 for the full set of countries, which is greater than the 5% critical value of a standard normal distribution. Greece is the country with the maximum individual $M_{\text{min,3}}$ and is thus removed from the panel. The persistence test is then re-applied to the remaining countries and the procedure continues until the panel test does not reject the null hypothesis. Table 7 reports those countries with a break in the inflation rates. The break dates, which are estimated using formula (42), are as follows: Greece (1998:1), Portugal (1988:4), Italy (1985:3), Japan (1978:2), France (1985:2), Spain (1996:1), UK (1982:1) and US (1982:2). The years 1991-94 mark a transition from a high inflation rate to a more moderate level in Greece. In 1995 the Bank of Greece adopts

\footnote{Chortareas and Kapetanios (2009) use their procedure to distinguish between stationary and non stationary series in an OECD exchange rate panel by applying Chang (2002), Im, Pesaran and Shin (2003) and Pesaran (2007) tests.}
a nominal exchange rate anchor as an important mechanism for disinflation (Karfakis et al., 2004) for the first time. As a result, inflation falls from 4% in 1997 to 2.4% in 1999. In France the change in regime of monetary policy and new wage bargaining policy in 1983 leads to a strong decline in the average yearly growth of the CPI from nearly 11.0% before the 1985 to 2.1% afterwards. In Italy there is also a drop in the inflation after 1985 (following the changes made to the income policy in 1984) falling from 8.0% to 4.2% in one year. In Japan in the mid-1970s substantial changes in inflation are recorded, peaking in 1974 and relapsing around the end of 1970s. The break date for Spain coincides with the year in which the government begins a monetary policy of the inflation targeting. Similar results are found regarding the break dates for the UK and the US. The shift in the UK occurs in the first quarter of 1982, whereas the US experiences a break in the second quarter of 1982 (similar findings are found in Halunga et al., 2009). Various factors contribute to the decline in UK inflation rate at the beginning of the 1980s: a restrictive monetary policy (the government attempts to gain in credibility for their deflationary policies by convincing economic agents that the reduction of inflation is the preeminent policy goal), a strong exchange rate, a sharp slowdown in economic activity, and the macroeconomic discipline implied by the Medium-Term Financial Strategy (Benati and Surico, 2008). For these reasons, the inflation rate in the UK falls steeply from 21.0% to 4.0% in the first four years of the 1980s (see Nelson, 2003). In the late 1970s, the US money stock is growing at an undesirably fast rate, the rate of inflation is accelerating, and the dollar is steadily depreciating in the foreign exchange markets. Consequently the Federal Reserve decides to take several actions, including a change in its operating procedures, in an attempt to reverse these trends. The new procedures remain in place until the end of 1982, when Federal Reserve Chairman Paul Volcker announces the intention to place less emphasis on the money stock for the time being (see Anderton, 1997). As a result, the inflation falls from 10.3% to 3.2% in 1981-1983.\(^6\)

\(^6\)Benati and Kapetanios (2002) show that after the Volcker disinflation
Conclusions

In this paper we present new panel tests for a change in persistence which are based on a modified time series version of the ratio-based statistics presented in Kim (2000), Kim et al. (2002), Busetti and Taylor (2004) and Harvey et al. (2006). The panel tests are used to test the null hypothesis of stationarity against the alternative of a change in persistence from I(0) to I(1), from I(1) to I(0) and when the direction is unknown. The null hypothesis may be rejected even if only one of the cross-section units shows a change in persistence since the null hypothesis assumes no change in persistence in any of the cross-section units. Therefore, the panel tests are sensitive to the selection of the cross-section units in the panel. Hence, following Chortareas and Kapetanios (2009), a procedure to identify the series which undergo changes in persistence is used.

Two sets of panel tests are proposed here. The first set is assigned to cross-sectionally independent panels. The asymptotic distributions of these tests are derived and are shown to be normally distributed. The second set uses the hypothesis of cross-section dependence and defactored data are then applied. On conducting a Monte Carlo experiment considerable increases in power with respect to the univariate changes in persistence tests are found. The new tests are applied to quarterly inflation rate of a panel of 19 OECD countries over the period 1972:2-2008:2. The results show evidence in favor of a change in persistence from I(1) to I(0) for a set of these countries.

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Appendix A: Mathematical Proofs

Proof of Theorem 2.1. By Theorem 3.1 in Kim (2000), it results
\[
\lim_{T \to +\infty} \hat{K}_{T,i}(\tau_i) = \frac{(1 - \tau_i)^{-2} \int_{\tau_i}^{1} B_i(r - \tau_i)^2 \, dr}{\tau_i^{-2} \int_0^{\tau_i} B_i(r)^2 \, dr},
\]
where \(\{B_i\}_{i=1}^{\infty}\) is a sequence of standard brownian bridges that are independent and identically distributed. Furthermore, by the hypotheses stated in Assumption 1, we have that
\[
\lim_{T \to +\infty} E \left[ H_j \left( \frac{(T - [T\tau_i])^{-2} \sum_{i=[T\tau_i]+1}^{T} S_{i,t}^{(1)}(\tau_i)^2}{\sum_{i=1}^{[T\tau_i]} S_{i,t}^{(0)}(\tau_i)^2} \right) \right] = \mu_j^{(Q)}, \quad i = 1, \ldots, N, \ j = 1, 2, 3;
\]
\[
\lim_{T \to +\infty} V \left[ H_j \left( \frac{(T - [T\tau_i])^{-2} \sum_{i=[T\tau_i]+1}^{T} S_{i,t}^{(1)}(\tau_i)^2}{\sum_{i=1}^{[T\tau_i]} S_{i,t}^{(0)}(\tau_i)^2} \right) \right] = \sigma_j^{(Q)}, \quad i = 1, \ldots, N, \ j = 1, 2, 3,
\]
with
\[
\mu_j^{(Q)} := E \left[ H_j \left( \frac{(1 - \tau_i)^{-2} \int_{\tau_i}^{1} B_i(r - \tau_i)^2 \, dr}{\tau_i^{-2} \int_0^{\tau_i} B_i(r)^2 \, dr} \right) \right], \quad i = 1, \ldots, N; \quad (A.1)
\]
and
\[
\sigma_j^{(Q)} := V \left[ H_j \left( \frac{(1 - \tau_i)^{-2} \int_{\tau_i}^{1} B_i(r - \tau_i)^2 \, dr}{\tau_i^{-2} \int_0^{\tau_i} B_i(r)^2 \, dr} \right) \right], \quad i = 1, \ldots, N. \quad (A.2)
\]
The continuous mapping theorem and the continuity of the functionals imply
\[
\lim_{T \to +\infty} H_j \left( \hat{K}_{T,i}(\tau_i) \right) = H_j \left( \frac{(1 - \tau_i)^{-2} \int_{\tau_i}^{1} B_i(r - \tau_i)^2 \, dr}{\tau_i^{-2} \int_0^{\tau_i} B_i(r)^2 \, dr} \right).
\]
The Central Limit Theorem guarantees that
\[
\lim_{N \to +\infty} \frac{1}{\sqrt{N\sigma_j^{(Q)}}} \sum_{i=1}^{N} \left[ H_j \left( \frac{(1 - \tau_i)^{-2} \int_{\tau_i}^{1} B_i(r - \tau_i)^2 \, dr}{N\tau_i^{-2} \int_0^{\tau_i} B_i(r)^2 \, dr} \right) - \mu_j^{(Q)} \right] \sim N(0, 1),
\]
and the Theorem is completely proved.

Proof of Theorem 2.2. By Theorem 3.1 in Busetti and Taylor (2004), it results
\[
\lim_{T \to +\infty} \hat{K}_{T,i} = \frac{\tau_i^{-2} \int_0^{\tau_i} [V_i^{**}(r)]^2 \, dr}{(1 - \tau_i)^{-2} \int_0^{1} [V_i^{**}(r)]^2 \, dr},
\]
where
\[
V_i^{**}(r) = V_i(r) - V_i(\tau_i) - (r - \tau_i)(1 - \tau_i)^{-1}(V_i(1) - V_i(\tau_i)).
\]
\( V_i^{***}(r) = V_i(r) - r\tau_i^{-1}V_i(\tau_i) \)

and

\[ V_i(r) = W_0(r) + c \left\{ \int_0^{\min(r,\tau_i)} W_c(s)ds + 1(r > \tau_i)[(r - \tau_i)W_c(\tau_i)] \right\}, \]

where \( W_0 \) and \( W_c \) are independent standard Wiener processes. Hypotheses stated in Assumption 1 imply

\[ \lim_{T \to +\infty} E\left[H_j\left(\frac{\left[T\tau_i\right]^{-2} \sum_{t=1}^{\left[T\tau_i\right]} S_{i,t}^{(0)}(\tau_i)^2}{(T - \left[T\tau_i\right])^{-2} \sum_{t=\left[T\tau_i\right]+1}^{T} S_{i,t}^{(1)}(\tau_i)^2}\right)\right] = \bar{\mu}_{j}^{(R)}, \quad i = 1, \ldots, N, \ j = 1, 2, 3; \]

\[ \lim_{T \to +\infty} \sqrt{V[H_j\left(\frac{\left[T\tau_i\right]^{-2} \sum_{t=1}^{\left[T\tau_i\right]} S_{i,t}^{(0)}(\tau_i)^2}{(T - \left[T\tau_i\right])^{-2} \sum_{t=\left[T\tau_i\right]+1}^{T} S_{i,t}^{(1)}(\tau_i)^2}\right)} = \bar{\sigma}_{j}^{(R)}, \quad i = 1, \ldots, N, \ j = 1, 2, 3, \]

where

\[ \bar{\mu}_{j}^{(R)} := E[H_j\left(\frac{\tau_i^{-2} f_0^{\tau_i} [V_i^{***}(r)]^2 dr}{(1 - \tau_i)^{-2} \int_{\tau_i}^{1} [V_i^{**}(r)]^2 dr}\right)], \quad i = 1, \ldots, N, \ j = 1, 2, 3, \quad (A.3) \]

and

\[ \bar{\sigma}_{j}^{(R)} = \sqrt{V[H_j\left(\frac{\tau_i^{-2} f_0^{\tau_i} [V_i^{***}(r)]^2 dr}{(1 - \tau_i)^{-2} \int_{\tau_i}^{1} [V_i^{**}(r)]^2 dr}\right)]}, \quad i = 1, \ldots, N, \ j = 1, 2, 3, \quad (A.4) \]

The continuous mapping theorem and the continuity of the functionals imply

\[ \lim_{T \to +\infty} H_j\left(K_{T,\tau_i}(\tau_i)\right) = H_j\left(\frac{\tau_i^{-2} f_0^{\tau_i} [V_i^{***}(r)]^2 dr}{(1 - \tau_i)^{-2} \int_{\tau_i}^{1} [V_i^{**}(r)]^2 dr}\right). \]

By Central Limit Theorem we have

\[ \lim_{N \to +\infty} \frac{1}{\sqrt{N} \bar{\sigma}_{j}^{(R)}} \sum_{i=1}^{N} \left[H_j\left(\frac{\tau_i^{-2} f_0^{\tau_i} [V_i^{***}(r)]^2 dr}{(1 - \tau_i)^{-2} \int_{\tau_i}^{1} [V_i^{**}(r)]^2 dr}\right) - \bar{\mu}_{j}^{(R)}\right] \sim N(0, 1), \]

and the Theorem is completely proved. ■

**Proof of Theorem 2.3.** It is a direct consequence of the Central Limit Theorem. ■
Proof of Theorem 3.1. By (29) we have

\[ \Delta \mu_{i,t} = 1(t > [\tau_i T])\eta_{i,t}. \] (A.5)

The break can be observed in a specified time of the sample period. By assumption, we have that \( \tau_i \) is an uniform random variable in \((0, 1)\). In our model we consider breaks in \([\tau_i T]\). Therefore, we do not lose of generality by assuming that \( \tau_i \) is discrete, and takes uniformly values in the set \( \Gamma := \{j/T\}_{j=0,...,T-1} \). This assumption is made throughout the proof. The distribution of \( \Delta \mu_{i,t} \) can be obtained using all the possible realizations of \( \tau_i \) and applying standard stochastic calculus techniques. For each \( x \in \mathbb{R} \), we have

\[
P(\Delta \mu_{i,t} \leq x) = \sum_{j=0}^{T-1} P(\Delta \mu_{i,t} \leq x \mid \tau_i = j/T) \cdot P(\tau_i = j/T) =
\]

\[
= \frac{1}{T} \cdot \sum_{j=0}^{T-1} P(\Delta \mu_{i,t}^{(j)} \leq x),
\] (A.6)

where

\[ \Delta \mu_{i,t}^{(j)} = \begin{cases} 
0, & \text{if } t \leq j; \\
\eta_{i,t}, & \text{if } t > j. 
\end{cases} \] (A.7)

For each \( j = 0, \ldots, T - 1 \), the definition of \( \eta_{i,t} \) implies that \( \Delta \mu_{i,t}^{(j)} \) is a stationary process with zero mean and finite variance (independent of \( t \)). Moreover, the expected value and variance of \( \Delta \mu_{i,t} \) can be obtained as a linear combination of expected values and variances of the \( \Delta \mu_{i,t}^{(j)}, j = 0, \ldots, T-1 \) using equation (A.6). Therefore, \( \Delta \mu_{i,t} \) has zero mean and finite variance (independent of \( t \)), and the theorem is completely proved. \( \blacksquare \)

Proof of Theorem 4.1 First of all, it should be noted that the statistics \( \Xi \) is a one-sided test. Denote the \( \alpha_T \)-critical value of the test \( \Xi \) as \( c_T \).

The algorithm does not stop as long as there exist any series with break in \( y_{(i)}_{1,t} \). The series with break are removed from the panel until no series with break are not contained in the set of series. Thus the following three statements are needed to be proved.
(I) Fixed $N$, then
\[
\lim_{T \to +\infty} P(\Xi > c_T) = 1,
\]
if $\Xi$ has been constructed on the set of series $y_{i(k),t}$, containing at least one series with break.

(II) Fixed $N$, then
\[
\lim_{T \to +\infty} P(\Xi > c_T) = 0,
\]
if $\Xi$ has been constructed on the set of series $y_{i(k),t}$, where no series of $y_{i(k),t}$ contains a break.

(III) If $y_{i(k),t}$ contains a series with break, then the maximum test $\xi_i$, conjugated with $\Xi$, corresponds to a series with a break, with probability approaching to 1.

We first prove (I). Under the alternative hypothesis, Kim (2000) shows that $\xi_i \sim O_p(1)$ or $\xi_i \sim O_p(T^2)$, depending on the considered model or time series statistics. This implies that $\Xi \sim O_p(\sqrt{N})$ or $\Xi \sim O_p(T^2\sqrt{N})$, even if $y_{i(k),t}$ contains only one series with break. Since $\sqrt{N} \to +\infty$ as well as $T^2\sqrt{N} \to +\infty$, we obtain that all the panel statistics denoted by $\Xi$ are consistent, for each $N$, even if $y_{i(k),t}$ contains only one series with break. We now need to prove that
\[
\frac{c_T}{T^2\sqrt{N}} \to 0
\]
to show the validity of (I). This is a direct consequence of the hypotheses and the convergence of $\xi_i$ to a functional of Brownian Motion, as $T \to +\infty$, following Chortareas and Kapetanios (2009).

We now show (II). If $y_{i(k),t}$ contains no series with break, then $\Xi = O_p(1)$. In this case $\alpha_T \to 0$ which implies that $c_T \to +\infty$, and (II) holds.

To complete the proof it should be noted that, if $G_m = 1$ and $G_n = 0$, then
\[
P(\xi_m > \xi_n) \to 1,
\]
for each $n, m$. Indeed, $\xi_m \to +\infty$ and $\xi_n = O_p(1)$, and then (III) holds true.

**Proof of Theorem 4.2** The proof is a direct consequence of Theorem 4.1 and Theorem 2 in Chortareas and Kapetanios (2009).
Table 1: Simulated moments for individual Kim(2000, 2002) and Busetti and Taylor tests (2004)

<table>
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<th>Q1</th>
<th>Q2</th>
<th>Q3</th>
<th>R1</th>
<th>R2</th>
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Mean - drift case

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Std. deviation - drift case
Table 2: Rejection frequencies min Panel Tests When the Model Does Not Allow for Cross Sectional Dependence. I(0) → I(1). Intercept case.

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Notes: Empirical sizes corresponding to a 5% nominal size.
Table 3: Rejection frequencies min Panel Tests When the Model Does Not Allow for Cross Sectional Dependence. I(1) $\rightarrow$ I(0). Intercept case.

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Notes: Empirical sizes corresponding to a 5% nominal size.
Table 4: Rejection frequencies of Defactored Modified Min Panel Tests. \( I(0) \rightarrow I(1) \). Intercept case.

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Notes: Empirical sizes corresponding to a 5% nominal size.
Table 5: Rejection frequencies of Defactored Modified Min Panel Tests. $I(1) \rightarrow I(0)$. Intercept case.

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<td>1.000</td>
<td>1.000</td>
<td>0.965</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
<td>1.000</td>
</tr>
</tbody>
</table>

Notes: Empirical sizes corresponding to a 5% nominal size.
Table 6: Panel test statistics. Inflation rate 1972:2-2008:2

<table>
<thead>
<tr>
<th>Panel Test</th>
<th>Cross section independence</th>
<th>Value</th>
<th>Cross section dependence</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>MQ_{min,1}</td>
<td>-3.43</td>
<td>MQ_{min,1}</td>
<td>-3.47</td>
<td></td>
</tr>
<tr>
<td>MQ_{min,2}</td>
<td>-2.63</td>
<td>MQ_{min,2}</td>
<td>-2.64</td>
<td></td>
</tr>
<tr>
<td>MQ_{min,3}</td>
<td>-3.80</td>
<td>MQ_{min,3}</td>
<td>-3.82</td>
<td></td>
</tr>
<tr>
<td>MR_{min,1}</td>
<td>17.11</td>
<td>MR_{min,1}</td>
<td>17.26</td>
<td></td>
</tr>
<tr>
<td>MR_{min,2}</td>
<td>19.96</td>
<td>MR_{min,2}</td>
<td>20.12</td>
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</tr>
<tr>
<td>MR_{min,3}</td>
<td>15.71</td>
<td>MR_{min,3}</td>
<td>17.14</td>
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<tr>
<td>MM_{min,1}</td>
<td>12.88</td>
<td>MM_{min,1}</td>
<td>13.00</td>
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<tr>
<td>MM_{min,2}</td>
<td>14.29</td>
<td>MM_{min,2}</td>
<td>14.41</td>
<td></td>
</tr>
<tr>
<td>MM_{min,3}</td>
<td>11.90</td>
<td>MM_{min,3}</td>
<td>13.21</td>
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</tr>
</tbody>
</table>

Table 7: Countries with break. Inflation rate 1972:2-2008:2

<table>
<thead>
<tr>
<th>MM_{min,3}</th>
<th>Country</th>
<th>Estimated break</th>
<th>MM_{min,3}</th>
<th>Country</th>
<th>Estimated break</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.97</td>
<td>Italy</td>
<td>1985:3</td>
<td>7.99</td>
<td>Italy</td>
<td>1985:3</td>
</tr>
<tr>
<td>5.14</td>
<td>France</td>
<td>1985:2</td>
<td>5.20</td>
<td>France</td>
<td>1985:2</td>
</tr>
<tr>
<td>4.10</td>
<td>Spain</td>
<td>1986:2</td>
<td>4.16</td>
<td>Spain</td>
<td>1996:1</td>
</tr>
<tr>
<td>2.62</td>
<td>UK</td>
<td>1982:1</td>
<td>2.67</td>
<td>UK</td>
<td>1982:1</td>
</tr>
<tr>
<td>1.72</td>
<td>US</td>
<td>1982:2</td>
<td>1.73</td>
<td>US</td>
<td>1982:2</td>
</tr>
</tbody>
</table>